Formally Correct Composition of Coordinated Behaviors Using Control Barrier Certificates *

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***Abstract*— In multi-robot systems, although the idea of behaviors allows for an efficient solution to low-level tasks, high-level missions can rarely be achieved by the execution of a single behavior. In contrast to this, a sequence of behaviors would provide the requisite expressiveness, but there are no a priori guarantees that the sequence is composable in the sense that the robots can actually execute it. In order to guarantee a provably correct composition of behaviors, Finite-Time Convergence Control Barrier Functions are introduced in this paper to guarantee the terminal configuration of one behavior is a valid initial configuration for the following one. Nominal control inputs prescribed by the behaviors are modified in a minimally invasive fashion, in order to establish the information-exchange network required by the following behavior. The effectiveness of the proposed composition strategy is validated on a team of mobile robots.**

I. INTRODUCTION

Teams of autonomous robots have a wide range of real-world applications including search and rescue [3], environmental surveillance [8] and space exploration [12]. Such high-level missions often require the team to accomplish a set of tasks through collective decision making and coordinated control. For example, the MAGIC 2010 competition [19] addressed a search and rescue scenario, where the robots were expected to complete a series of tasks including exploration, sensor fusion, target localization and human-robot interaction.

Given the complexity of these high-level missions, it is natural to decompose them into a sequence of dedicated coordinated tasks and design *behaviors* which achieve each of the tasks independently. In the context of multi-robot systems, this decomposition is particularly advantageous because (i) there has been a variety of coordinated behaviors developed for elementary tasks such as flocking [17], rendezvous [18], formation control [7], etc, and (ii) the computational burden would quickly become intractable if the complete mission is addressed as a single, highly complex search problem. Hence, it is favorable to leverage well-established and provably correct coordinated behaviors to achieve complicated missions by composing these behaviors.

The main challenge associated with such composition is that most behaviors rely on a set of underlying assumptions to function as desired. One of the common assumptions for coordinated behaviors is the presence of sufficiently rich proximity-based information-exchange networks. For example, in order to have the robots rendezvous, the information-exchange network should remain connected [17]. Therefore, in order to complete the high-level mission through a sequence of behaviors, not only should each of the single behaviors be correct, but their assumptions should also be satisfied both at the starting time of the behaviors and throughout their execution.

The decomposition of high-level missions into simple tasks is done routinely for single robots, e.g., [2], even with formal guarantees on the overall performance [6]. The idea to decompose complex missions has also been explored in the context of multi-robot systems using models such as Petri Nets [10], finite state automata [13], stochastic processes [16], temporal logic specifications [11], [21], embedded graph grammars [9], [22], and graph process specifications [23]. However, that body of work is typically task

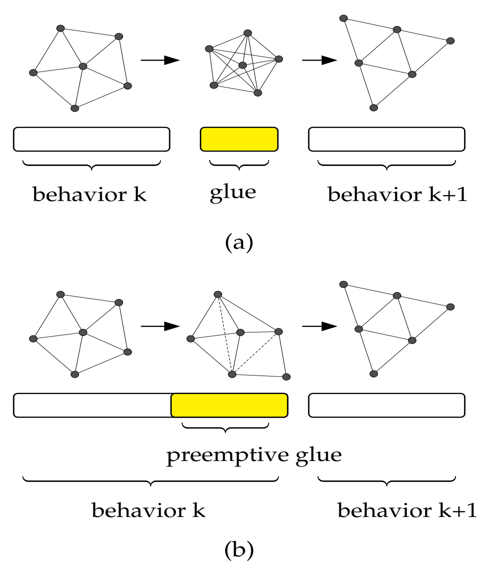

Fig. 1: Examples of different approaches in behavior composition. In (a), a hand-crafted transition behavior is used as a "glue" to ensure the communication graph structure is established for the following behavior [23]. In (b), the approach presented in this paper is shown, where each of the individual behaviors is modified in order to provide a "preemptive glue" for the composition.

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specific. But more importantly, it focuses on conditions and formalisms for characterizing when a sequence of behaviors can be composed together. Although a subset of this work proposes transition behaviors to be inserted between behavior pairs that are not composable [23], such hand-crafted transition behaviors are not always desirable because they are often elaborate and time-consuming. In this paper, we take a different view of this problem by *actively modifying the behaviors* so that they are provably composable, as illustrated in Fig. 1.

In this paper, we focus on providing formal guarantees on composition of coordinated behaviors. Namely, ensuring that once a behavior terminates, the underlying assumptions for the upcoming behavior are satisfied. We develop a variant of control barrier functions (CBFs) [1], [5] called *Finite-Time Convergence Control Barrier Functions*. By encoding the assumptions for each behavior into constraints on Finite-Time Convergence Control Barrier Functions, we can synthesize controllers that formally guarantee that (i) if the assumptions required by a behavior are satisfied at some point, they will remain satisfied thereafter. On the other side, (ii) if the assumptions are not satisfied initially, they will be satisfied within finite time.

The main contribution of this paper is two-fold. Firstly, a variant of control barrier functions is introduced to ensure that the states evolve into the desired subset of the state space within finite time. Secondly, a framework based on control barrier functions is presented to actively modify the behaviors so that the resulting composition of behaviors proves formally correct.

The rest of this paper is organized as follows. Section II formulates the behavior composition problem. Section III introduces finite-time convergence control barrier functions and discusses their application in the context of the behavior composition problem. Section IV proposes a framework for composition of behaviors with theoretical guarantees. At last, experimental results and conclusions are in Section V and Section VI, respectively.

II. Composition of Coordinated Behaviors

Consider a homogeneous robotic team consisting of N mobile robots with integrator dynamics in an uncluttered 2-dimensional environment, i.e., $\dot{x}_i = u_i$, where $x_i \in \mathbb{R}^2$, $u_i \in \mathbb{R}^2$ are the state vector and control input for robot i. Let $x = [x_1^\top, \dots, x_N^\top]^\top$ and $u = [u_1^\top, \dots, u_N^\top]^\top$ denote the joint state vector and control input for the robotic team, respectively.

The communication graph structure of the robotic team is defined as $\mathscr{G}(t) = (V, E(t))$, where $V = \{1, 2, \dots, N\}$ is the vertex set consisting of N mobile robots and $E(t)$ is the set of edges at time t. We assume a proximity-based communication graph, namely, the presence of an edge (i, j) indicates that robot i and robot j are within a communication distance of D_c, i,e.,

$$(i,j) \in E(t) \iff \|x_i(t) - x_j(t)\| \leq D_c, \tag{1}$$

where the communication graph is time-varying due to the motion of robots.

The high-level mission of the robotic team is provided by the human operator before the operation. The mission is specified through a motion sequence σ, consisting of M coordinated behaviors, e.g., flocking [17], rendezvous [18], formation control [7], etc. We assume that a rich enough communication graph is a sufficient condition for each coordinated behavior to function as desired, which is a common requirement for decentralized coordinated behaviors. For example, a strongly connected communication graph guarantees that the flocking and rendezvous behaviors can be achieved through consensus [18], and a rigid graph is required for a formation control law to function correctly [15].

We use notations similar to the motion description language [14] to specify the sequence of behaviors, namely,

$$\sigma = (\mathscr{U}_1, \mathscr{G}_1, \tau_1), \cdots, (\mathscr{U}_M, \mathscr{G}_M, \tau_M), \tag{2}$$

where each coordinated behavior is represented by a tuple $(\mathscr{U}_k, \mathscr{G}_k, \tau_k)$. Here, $\mathscr{U}_k$, $\mathscr{G}_k$, and τ_k are the coordinated controller, required communication graph structure, and starting time of behavior k, respectively. The required graph structure $\mathscr{G}_k$ is defined through (V, E_k). To ensure that behavior k is executed correctly, the required graph $\mathscr{G}_k$ should remain a spanning sub-graph of the actual communication graph $\mathscr{G}(t)$ throughout the execution of behavior k, i.e., $(i, j) \in E(t)$, $\forall (i, j) \in E_k, t \in [\tau_k, \tau_{k+1})$. A segment of an example motion sequence is shown in Fig. 2.

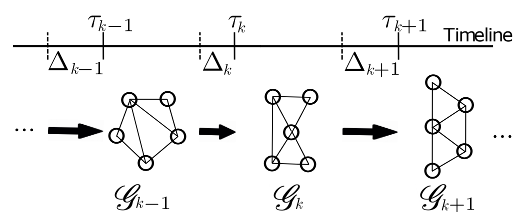


Fig. 2: A segment of a sequence of formation control behaviors for a team of five robots. Each formation control behavior k starts at τ_k and requires a communication graph structure $\mathscr{G}_k$ in order to function properly. The proposed framework assigns a preparation time interval of length Δ_k, during which the preceding behavior is modified so as to establish the required communication graph structure by the time of τ_k.

Based on the aforementioned assumptions, any two adjacent behaviors can be composed together through assembling a complete communication graph during the transition, since the required communication graph for any behavior is a spanning sub-graph of a complete graph. In fact, [23] makes use of this observation and inserts hand-crafted transition behaviors such that a complete communication graph is formed during the transition, as is shown in Fig. 1a. In this paper, however, we modify the behaviors during their execution in a way that the behaviors can be composed in a less invasive manner.

Note that although we consider the communication graph structure as the only type of constraints required for the functioning of a behavior, one must acknowledge that different types of constraints could be considered as well, e.g. the position of a robot in the global frame of reference, or the relative bearings. In fact, the technique developed in this paper is rather general and can be extended to other constraints as long as they can be encoded through control barrier functions introduced in Section III.

In summary, to ensure that the composition of behaviors is valid, we need to make sure that each behavior k starts with the required graph structure $\mathscr{G}_k$ assembled, i.e., $E_k \subset E(\tau_k)$, and the required communication graph structure is maintained throughout the execution. To this end, we develop *Finite-Time Convergence Control Barrier Functions* to provide theoretical guarantees on composition of behaviors.

III. Finite-Time Convergence Control Barrier Functions

In this section, we present *Finite-Time Convergence Control Barrier Functions* that are essential for valid behavior composition. We will start with the general formulation that is applicable to any control affine system. Then we will introduce the application of Finite-Time Convergence Control Barrier Functions on establishing a required communication graph structure within finite time.

Consider a nonlinear control system in control affine form,

$$\dot{x} = f(x) + g(x)u, \tag{3}$$

where $x \in \mathscr{D} \subset \mathbb{R}^n$ and $u \in U \subset \mathbb{R}^m$, f and g are locally Lipschitz continuous. For the sake of simplicity, we assume that (3) is forward complete, i.e. solutions $x(t)$ are defined for all $t \geq 0$.

Without loss of generality, let the desired set for the state of the system be defined as a super-level set of a continuously differentiable function $h(x)$,

$$\mathscr{C} = \{x \in \mathbb{R}^n : h(x) \geq 0\}. \tag{4}$$

In order to guarantee the correct functioning of each behavior, the corresponding constraints must be satisfied at the behavior's starting time, and throughout its entire duration. This, in the context of controller design, can be translated as follows. (i) For trajectories that start outside $\mathscr{C}$, they should enter $\mathscr{C}$ within a specific finite interval of time. (ii) The controller should render the desired set forward invariant, i.e., if solution of (3) starts in the set $\mathscr{C}$, it will stay in $\mathscr{C}$ for all future time.

Control barrier functions [1], [5] are Lyapunov-like functions that are used to provably ensure the forward-invariance of a desired set. Therefore, by their definition, the use of control barrier functions automatically guarantees the satisfaction of the forward invariance requirement. In addition to the forward invariance property, *Zeroing control barrier functions* [24], a variant of control barrier functions, can be used to guarantee that the state will converge to the boundary of the desired set asymptotically when its initial condition lays outside of the desired set. However, in the context of this paper, since the behavior composition problem is defined by a sequence of predefined starting time of behaviors, asymptotic convergence guarantees are not sufficient.

In order to provide theoretical guarantees for both forward invariance and finite-time convergence, we propose a new variant of control barrier functions called *Finite-Time Convergence Control Barrier Functions*.

Definition III.1. *Given a dynamical system (3) and the set $\mathscr{C}$ defined by (4) with a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$, if there exist real parameters $\rho \in [0,1), \gamma > 0$, and a set $\mathscr{C} \subseteq \mathscr{D} \subset \mathbb{R}^n$ such that, for all $x \in \mathscr{D}$,*

$$\sup_{u \in U} \left[L_f h(x) + L_g h(x) u + \gamma \cdot \operatorname{sign}(h(x)) \cdot |h(x)|^\rho \right] \geq 0, \tag{5}$$

then the function h is called a Finite-time Convergence Control Barrier Function (FCBF) defined on the set $\mathscr{D}$.

Given a FCBF h, the set of feasible control inputs is,

$$\begin{aligned} K(x) = \big\{ & u \in U : \\ & L_f h(x) + L_g h(x) u + \gamma \cdot \operatorname{sign}(h(x)) \cdot |h(x)|^\rho \geq 0 \big\}. \end{aligned} \tag{6}$$

Proposition III.1. *Given a set $\mathscr{C} \subset \mathbb{R}^n$ associated with a FCBF $h(x)$ defined on $\mathscr{D}$ with $\mathscr{C} \subseteq \mathscr{D} \subset \mathbb{R}^n$, and parameters $\rho \in [0,1), \gamma > 0$, any continuous controller $u \colon \mathscr{D} \to U$ such that $u \in K(x)$ for the system (3) renders the set $\mathscr{C}$ forward invariant. Moreover, given the initial state $x_0 \in \mathscr{D} \setminus \mathscr{C}$, any continuous controller $u \colon \mathscr{D} \to U$ such that $u \in K(x)$ for the system (3) drives the state $x(t)$ to $\mathscr{C}$ within finite time $T = \frac{1}{\gamma(1-\rho)} |h(x_0)|^{1-\rho}$.*

Proof. Consider the Lyapunov candidate function $V(x) = \max\{-h(x), 0\}$. The function satisfies, $V(x) = 0$ for $x \in \mathscr{C}$, $V(x) > 0$ for $x \in \mathscr{D} \setminus \mathscr{C}$, and $\dot{V}(x(t)) \leq -\gamma \cdot V^\rho(x(t))$ for $t > 0$.

Assume that $x_0 \in \mathscr{C}$. By the comparison lemma [4], $V(x(t)) = 0$ for all $t > 0$. Therefore, $u \in K(x)$ renders the set $\mathscr{C}$ forward invariant. Assume that $x_0 \in \mathscr{D} \setminus \mathscr{C}$. According to [4], the state x will converge to the set $\mathscr{C}$ in finite time within $T = \frac{1}{\gamma(1-\rho)} |h(x_0)|^{1-\rho}$. □

To sum up, we can use Finite-Time Convergence Control Barrier Functions to design controllers ensuring that the system will enter the desired set within finite time and stay in the set thereafter.

A. Communication Graph Barrier Certificates

Here we discuss how to establish a communication graph structure within finite time using Finite-Time Convergence Control Barrier Functions. As discussed in Section II, assume that the robotic team is required to establish a communication graph $\mathscr{G}_k = (V, E_k)$ by time τ_k. For each single edge $(i, j) \in E_k$, the edge requirement can be encoded into a set $\bar{\mathscr{C}}_{ij}$ defined by a function $\bar{h}_{ij}$, where,

$$\begin{aligned} \bar{\mathscr{C}}_{ij} &= \{x \in \mathbb{R}^{2N} : \bar{h}_{ij}(x) \geq 0\}, \\ \bar{h}_{ij}(x) &= D_c^2 - (x_i - x_j)^\top (x_i - x_j). \end{aligned} \tag{7}$$

Then, the overall communication graph requirement for behavior k can be encoded into a single set $\bar{\mathscr{C}}_k$ as the intersection of $\bar{\mathscr{C}}_{ij}$ for each $(i, j) \in E_k$,

$$\bar{\mathscr{C}}_k = \bigcap_{(i,j) \in E_k} \bar{\mathscr{C}}_{ij}. \tag{8}$$

If the states enter $\bar{\mathscr{C}}_{ij}$ before τ_k for all $(i,j) \in E_k$, the states will enter $\bar{\mathscr{C}}_k$ before time τ_k. Hence, if the time derivative of $\bar{h}_{ij}(x)$ satisfies,

$$\dot{\bar{h}}_{ij}(x) + \gamma \cdot \mathrm{sign}(\bar{h}_{ij}(x)) \cdot |\bar{h}_{ij}(x)|^\rho \geq 0, \tag{9}$$

for all $(i,j) \in E_k$, the required communication graph will be formed within finite time. Therefore, by assembling all the communication graph constraints in (9) together, we get the communication graph barrier certificates $\bar{\mathscr{B}}_k$ for behavior k,

$$\begin{aligned} \bar{\mathscr{B}}_k = \{u \in \mathbb{R}^{2N} : \dot{\bar{h}}_{ij}(x) + \gamma \cdot \mathrm{sign}(\bar{h}_{ij}(x)) \cdot |\bar{h}_{ij}(x)|^\rho \geq 0, \\ \forall (i,j) \in E_k\}. \end{aligned} \tag{10}$$

By restricting the control input u to the certificates $\bar{\mathscr{B}}_k$, the robotic team is guaranteed to assemble the required communication graph structure $\mathscr{G}_k$ within finite time. This result is summarized in the following theorem.

Theorem III.2. *Given a required communication graph structure $\mathscr{G}_k = (V, E_k)$ and a robotic team with initial state x_0, any controller $u(x) \in \bar{\mathscr{B}}_k(x)$ can assemble the required graph $\mathscr{G}_k$ within finite time duration*

$$T_k = \max_{\{(i,j) \in E_k : x_0 \notin \bar{\mathscr{C}}_{ij}\}} \left\{ \frac{1}{\gamma(1-\rho)} |\bar{h}_{ij}(x_0)|^{1-\rho} \right\}.$$

Proof. Consider any pair of robots i and j with $(i,j) \in E_k$. Since the controller $u(x)$ satisfies the pairwise communication graph barrier constraint in (9), according to Proposition III.1, if $x_0 \notin \bar{\mathscr{C}}_{ij}$, the system will be driven into $\bar{\mathscr{C}}_{ij}$ within time duration $T_{ij} = \frac{1}{\gamma(1-\rho)} |\bar{h}_{ij}(x_0)|^{1-\rho}$. Also according to Proposition III.1, if $x_0 \in \bar{\mathscr{C}}_{ij}$, the system will stay within $\bar{\mathscr{C}}_{ij}$. Note that in this case $\bar{h}_{ij}(x_0) \geq 0$ and $T_{ij} = 0$.

Since every set $\bar{\mathscr{C}}_{ij}$ will be reached within time duration $T_{ij} = \frac{1}{\gamma(1-\rho)} |\bar{h}_{ij}(x_0)|^{1-\rho}$ if $x \notin \bar{\mathscr{C}}_{ij}$, the overall communication graph structure requirement encoded by $\bar{\mathscr{C}}_k$ will be satisfied within finite time duration

$$T_k = \max_{\{(i,j) \in E_k : x_0 \notin \bar{\mathscr{C}}_{ij}\}} \left\{ \frac{1}{\gamma(1-\rho)} |\bar{h}_{ij}(x_0)|^{1-\rho} \right\}.$$

□

According to Theorem III.2, we just need to activate the barrier certificates for an appropriate time interval Δ_k before τ_k, where Δ_k needs to satisfy $T_k \leq \Delta_k \leq \tau_k - \tau_{k-1}$. Then, the required communication graph structure $\mathscr{G}_k$ is guaranteed to be constructed before the behavior starting time τ_k. One may notice that the parameters γ and ρ need to be selected accordingly so that such an Δ_k exists. The design strategies for choosing these parameters are further discussed in Section V-C.

IV. The Behavior Composition Framework

In this section, we introduce a behavior composition framework anchored by Finite-Time Convergence Control Barrier Functions discussed in the previous section. The main feature of our framework is that there are no explicit transition modes inserted between behaviors. When executing behavior k, the robots also need to prepare for behavior $k+1$, i.e., all the edges in $\mathscr{G}_{k+1}$ for behavior $k+1$ need to be established in the communication graph before time τ_{k+1}. We present a framework consisting of a sequence of optimization-based controllers constrained to communication graph barrier certificates. Each of the optimization-based controllers is a minimally invasive controller, i.e. each behavior will be modified only when required by its composition with the following behavior.

A. Optimization-based Controller

We propose an optimization-based controller that respects the communication graph barrier certificates $\bar{\mathscr{B}}_k(x)$ while keeping the modification to each behavior minimal. We assume that a nominal controller $\hat{u}_{i,k}$ for each robot i and behavior k is designed with existing control techniques and is given while executing the behaviors.

For each behavior k, we divide the time interval during which behavior k is executed into two phases: before activating $\bar{\mathscr{B}}_{k+1}$ and after activating $\bar{\mathscr{B}}_{k+1}$, i.e., before and after the robots start to get prepared for behavior $k+1$. Let Δ_{k+1} be the preparation time for behavior $k+1$. Then the first phase is formally defined over $t \in [\tau_k, \tau_{k+1} - \Delta_{k+1})$, and the second phase is defined over $t \in [\tau_{k+1} - \Delta_{k+1}, \tau_{k+1})$.

First, consider the phase when the robots are not preparing for the next behavior. In this case, the robots only need to maintain the communication graph structure required by the current behavior k. Therefore, the goal is to make the actual control input u_i as close to the nominal control input $\hat{u}_{i,k}$ as possible while staying inside the certified space $\bar{\mathscr{B}}_k$,

$$\begin{aligned} u^* = \quad & \arg\min_u \sum_{i=1}^N \|u_i - \hat{u}_{i,k}\|^2 \\ \text{s.t.} \quad & \dot{\bar{h}}_{ij}(x) + \gamma \cdot \mathrm{sign}(\bar{h}_{ij}(x)) \cdot |\bar{h}_{ij}(x)|^\rho \geq 0,\ \forall (i,j) \in E_k. \end{aligned} \tag{11}$$

Here the forward invariance property of Finite-Time Convergence Control Barrier Functions is used to ensure the required communication graph structure is preserved.

In the second phase, the robots not only need to maintain the communication graph required by behavior k, they also need to actively assemble the communication graph required by the next behavior. Therefore, the control input must satisfy an additional constraint provided by $\bar{\mathscr{B}}_{k+1}$,

$$\begin{aligned} u^* = \quad & \arg\min_u \sum_{i=1}^N \|u_i - \hat{u}_{i,k}\|^2 \\ \text{s.t.} \quad & \dot{\bar{h}}_{ij}(x) + \gamma \cdot \mathrm{sign}(\bar{h}_{ij}(x)) \cdot |\bar{h}_{ij}(x)|^\rho \geq 0,\ \forall (i,j) \in E_k, \\ & \dot{\bar{h}}_{ij}(x) + \gamma \cdot \mathrm{sign}(\bar{h}_{ij}(x)) \cdot |\bar{h}_{ij}(x)|^\rho \geq 0,\ \forall (i,j) \in E_{k+1}. \end{aligned} \tag{12}$$

In this case, the forward invariance property is used to ensure the required communication graph structure $\mathscr{G}_k$ is maintained while the finite-time convergence property is used to ensure the required communication graph structure $\mathscr{G}_{k+1}$ is formed before behavior $k+1$ starts.

In summary, during the execution of behavior k, the optimization-based behavior is given by,

$$u = \begin{cases} \arg\min_{u \in \bar{\mathscr{B}}_k} \sum_i \|u_i - \hat{u}_{i,k}\|^2, & \text{if } t \in [\tau_k, \tau_{k+1} - \Delta_{k+1}), \\ \arg\min_{u \in \bar{\mathscr{B}}_k \cap \bar{\mathscr{B}}_{k+1}} \sum_i \|u_i - \hat{u}_{i,k}\|^2, & \text{if } t \in [\tau_{k+1} - \Delta_{k+1}, \tau_{k+1}). \end{cases} \tag{13}$$

The minimally invasive control input can be obtained as a solution to the above Quadratic Programming (QP) problem since the objective in (13) is quadratic in u and the constraints are linear in u. QPs can be solved very efficiently, which enables real-time implementation of this framework.

B. Overarching Constraints

In addition to the communication graph requirements given by the composability of behaviors, overarching constraints that need to be satisfied throughout the execution of the sequence might be considered as well. For example, it is important to ensure that the trajectories are collision-free and the speeds of the robots are within their limits. In this section, we will use collision-free constraints and speed limits as examples to show how to incorporate overarching constraints into the proposed behavior composition framework.

To produce collision-free motions, we encode collision avoidance as an additional constraint in the optimization-based controller. In this case, we need the collision-free constraints be linear with respect to the control input to preserve the efficiency of quadratic programming. As an example, we consider the safety barrier certificates developed in [5]. We refer the readers to [5] for detailed derivations and proofs of the safety barrier certificates.

Consider any pair of robots i and j, a pairwise safe set can be defined as,

$$\begin{aligned} \mathscr{C}_{ij} &= \{x \in \mathbb{R}^{2N} : h_{ij}(x) \geq 0\}, \\ h_{ij}(x) &= (x_i - x_j)^\top (x_i - x_j) - D_s^2, \ \forall i > j, \end{aligned} \tag{14}$$

where $D_s > 0$ is the safety distance that every pair of robots need to maintain. The forward invariance of this pairwise safe set can be ensured by the following constraint on control barrier function $h_{ij}(x)$,

$$\dot{h}_{ij}(x) + \alpha \cdot h_{ij}^3(x) \geq 0, \tag{15}$$

where $\alpha > 0$ is a design parameter which serves a role similar to the parameter γ in (9). To ensure collision-free trajectories, all pairwise conflicts need to be be addressed. Thus, the team-level collision-free set can be written as,

$$\mathscr{C}_s = \bigcap_{\{i,j \in V : i > j\}} \mathscr{C}_{ij}. \tag{16}$$

Assembling both the communication graph barrier constraints and safety barrier constraints together, we can form the certified admissible control space $\mathscr{B}_k(x)$,

$$\begin{aligned} \mathscr{B}_k(x) = \bar{\mathscr{B}}_k(x) \cap \\ \{u \in \mathbb{R}^{2N} : \dot{h}_{ij}(x) + \alpha \cdot h_{ij}^3(x) \geq 0, \ \forall i > j\}. \end{aligned} \tag{17}$$

By constraining the control input u within the certified space $\mathscr{B}_k(x)$, the robots will both stay collision-free and form the required graph $\mathscr{G}_k$ within finite time.

Theorem IV.1. *Given a required communication graph $\mathscr{G}_k = (V, E_k)$ and a robotic team with initial state $x_0 \in \mathscr{C}_s$, any continuous controller $u(x)$ that satisfies the communication graph and safety certificates $\mathscr{B}_k$ wil render the team collision free and form the required graph $\mathscr{G}_k$ within finite time duration $T_k = \max_{\{(i,j) \in E_k : x_0 \notin \bar{\mathscr{C}}_{ij}\}} \{\frac{1}{\gamma(1-\rho)} \bar{h}_{ij}(x_0)^{1-\rho}\}$.*

Proof. Since $u(x) \in \mathscr{B}_k(x) \subseteq \bar{\mathscr{B}}_k(x)$, the robots will form the required communication graph structure $\mathscr{G}_k$ within finite time duration T_k according to Theorem III.2. The team is guaranteed collision-free according to the forward invariance property of the control barrier functions [5]. □

Additionally, we can assume that each robot i can not reach a speed that is beyond its speed limit $v_{i,\max}$ in each dimension. Hence, we need $u \in U$, where,

$$U = \{u \in \mathbb{R}^{2N} : \|u_i\|_\infty \leq v_{i,\max}, \ \forall i\}. \tag{18}$$

Consequently, the optimization-based controller respecting safety constraints and speed limits is given by,

$$u = \begin{cases} \underset{u \in \mathscr{B}_k \cap U}{\arg\min} \sum_i \|u_i - \hat{u}_{i,k}\|^2, & \text{if } t \in [\tau_k, \tau_{k+1} - \Delta_{k+1}), \\ \underset{u \in \mathscr{B}_k \cap \mathscr{B}_{k+1} \cap U}{\arg\min} \sum_i \|u_i - \hat{u}_{i,k}\|^2, & \text{if } t \in [\tau_{k+1} - \Delta_{k+1}, \tau_{k+1}). \end{cases} \tag{19}$$

Note that since all the constraints in (19) can be written as linear constraints, the resulting controller can still be solved efficiently through quadratic programming in real-time.

V. Experimental Results

In this section, we validate the behavior composition framework through both numerical simulations as well as robotic implementations. Towards the end of this section, we discuss the choice of parameters for the Finite-Time Control Barrier Functions in practice.

A. Simulation Results

We evaluate the framework presented in this paper by simulating a team of 6 agents performing an environmental exploration task. In particular, the robots were tasked with reaching a region of interest, denoted by the blue ellipse, and entering it. As discussed in Section II, the overall mission is represented as a sequence of behaviors to be performed sequentially by the robots. With reference to Fig. 3, the sequence of behaviors was composed by an initial lattice formation control, where robots gathered in the proximity of their initial positions forming a regular lattice. After that, a leader-follower behavior was employed, where agent 1 drove the team to a pre-assigned point close to the region of interest. Once the region of interest had been approached, a formation control behavior moved the agents into a cross-shaped formation. In the final stage, through leader-follower behavior, agent 6 moved the team inside the region. The colored bar at the bottom of each figure represents the time progress of the entire mission; each behavior is represented by a light color, while darker colors denote the time during which the robots were preparing for the following behavior.

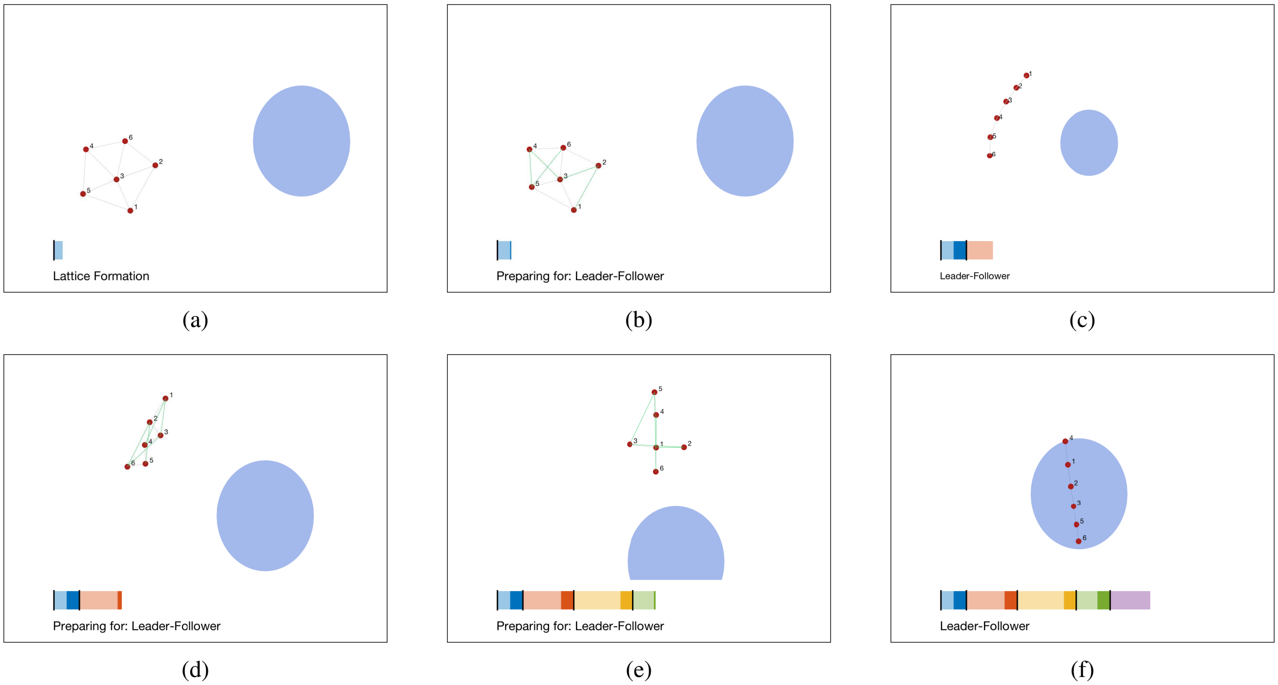


Fig. 3: A sequence of behaviors representing an environmental exploration task. A team composed by six robots was tasked with investigating a region of interest, denoted by the blue ellipse. Robots initially gathered under a lattice formation control behavior; after that a sequence of leader-follower behaviors and formation control behaviors moved the agents close to the region of interest and prepared the team for entering it. The mission was concluded with a final leader-follower behavior, where agent 6 led the team inside the region. In the figure, gray lines denote the edges of the graph required by the current behavior, while green lines represent the additional edges required by the next behavior.

B. Robotic Implementations

In this section we present results from the experiment conducted on the Robotarium, a remotely accessible robotic test-bed [20]. In the experiment, the robotic team executed a sequence of behaviors consisting of a cyclic pursuit behavior, a lattice formation (Formation 1), a line formation (Formation 2) and a square formation (Formation 3). The correctness of behavior composition was formally guaranteed by the framework introduced in this paper whereas safety of the robots was encoded by the safety barrier certificates [5] as an overarching constraint. Fig. 4 shows the snapshots taken during the experiment. The robots were initialized with a cycle formation and executed the cyclic pursuit behavior. Next, the robots modified the cyclic pursuit behavior to establish the required edges for the lattice formation (Fig. 4a), and then formed the lattice formation (Fig. 4b). After that the robot achieved a line formation and a square formation successively in the similar manner (Fig. 4c–4f). As is shown in Fig. 4, the edges required by each behavior (red) remained connected throughout the execution, and the additional edges required by the following behavior (blue) were always established during the preparation period.

C. Discussions

Here, we discuss some practical considerations when implementing the framework on robotic teams, including how to determine preparation time duration Δ_k for each behavior, and how to choose the parameters γ and ρ for the control barrier functions accordingly.

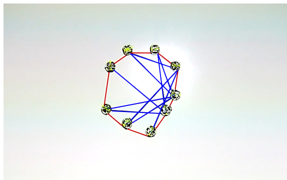
There are two fundamental ways to design preparation time Δ_k for each coordinated behavior k. The first one is named as *event-triggered* where a fixed preparation time Δ_k is specified ahead of time. The finite-time convergence control barrier certificates are automatically activated at $t = \tau_k - \Delta_k$, where the parameters γ and ρ in the control barrier certificates are computed such that,

$$T_k = \max_{\{(i,j)\in E_k : x(t)\notin \bar{\mathscr{C}}_{ij}\}} \left\{ \frac{1}{\gamma(1-\rho)} |\bar{h}_{ij}(x(t))|^{1-\rho} \right\} \leq \Delta_k. \quad (20)$$

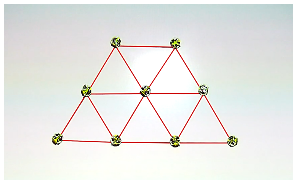
The second approach is called *self-triggered*. In this case, we specify fixed γ and ρ, and at each time, we check whether the finite-time convergence control barrier certificates need to be activated by the worst case scenario in the domain $\mathscr{D}$, i.e., we activate the finite-time convergence control barrier certificates at time t when,

$$\tau_k - t \leq \max_{x\in\mathscr{D}} \max_{\{(i,j)\in E_k : x\notin \bar{\mathscr{C}}_{ij}\}} \left\{ \frac{1}{\gamma(1-\rho)} |\bar{h}_{ij}(x)|^{1-\rho} \right\} + \varepsilon, \quad (21)$$

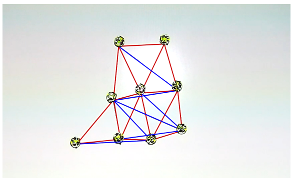
where $\varepsilon > 0$ is a small positive scalar. Note that this way of designing the preparation time often requires additional knowledge about the domain of the joint state $\mathscr{D}$, for example, knowing that $\mathscr{D}$ is bounded or compact.



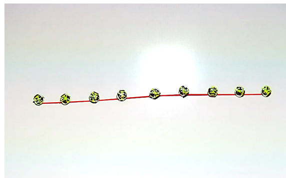
(a) Preparing for Formation 1



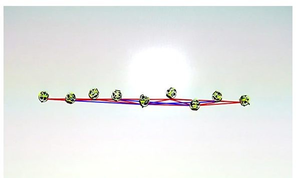
(b) Formation 1



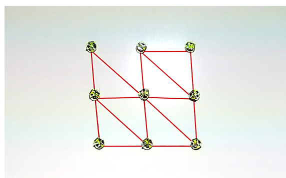
(c) Preparing for Formation 2



(d) Formation 2



(e) Preparing for Formation 3



(f) Formation 3

Fig. 4: Experimental results for a team of nine mobile robots executing a sequence of behaviors on the Robotarium testbed. The graphics were superimposed on the robot arena by an overhead projector whereas the colors of the graphics are augmented in the figures for the purpose of visualization. The edges required by each behavior remained connected (red) throughout its execution, and the additional edges (blue) required by the following behavior were established during the preparation time. Robots used in the experiments appear different from [20] because a different tracking system was used.

VI. Conclusions

The paper presented a framework that provides theoretical guarantees on composition of coordinated behaviors for teams of autonomous robots. To achieve provably correct composition, Finite-Time Convergence Control Barrier Functions were proposed so that the robots can assemble the required communication graph structure within a specified finite interval of time. The control barrier function-based framework actively modifies the behaviors in the minimally invasive way so that the requirements for each single behavior are satisfied when executed in sequence. Simulation results and experimental results on a team of mobile robots validated the effectiveness of the proposed behavior composition framework.

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